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Environmental Modeling Breakthrough: New Mathematical Solutions for Climate Impact Assessment

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Abstract

Over the past 20 years, China's construction industry is responsible for 64.6 percent of all China's total emission from 1991 to 2010, and its growth rate per year averages 5.7 percent. These figures are ominous enough to underscore the importance of environmental modeling needed for the determination and management of climate impact in the various industries. Breakthroughs in mathematical models are replacing our current way of assessing environmental impacts. Importantly, when used for the analysis of Environmental Impact Assessment (EIA) of the iron ore opencast mines of Iran, these models sought to identify potential unsustainability concerns in EIA that might have been overlooked. Additionally, such advanced modeling tools are supporting us in tackling severe environmental issues, including the carbon footprint and the assessment of air pollution. In this paper we discuss the newest evolution in environmental modeling, in particular mathematical solutions that are big changes in the future of climate impact assessments. The technical inputs of this analysis include innovative optimization techniques, practical engineering applications and sustainable technologies to create the future of environmental engineering. Using this systematic evaluation, we will understand how these mathematical approaches are delivering more fact based and actionable insights for making environmental decisions.

Keywords: Climate Impact Assessment; Environmental Modeling; Mathematical Solutions; Predictive Analysis; Sustainability Science

Abbreviations: EIA: Environmental Impact Assessment CNN-LSTM: Convolutional Neural Network – Long Short-Term Memory AI: Artificial Intelligence CMP: Coupled Model Intercomparison Project CFD: Computational Fluid Dynamics ODE: Ordinary Differential Equation RMSE: Root Mean Square Error MAE: Mean Absolute Error BIM: Bayesian Inference Model EMVL: Environmental Modeling and Visualization Laboratory

1. Evolution of Mathematical Models in Environmental Assessment

Among the many models which have emerged as indispensable tools for theoretical and applied ecology, those that characterize system level dynamics and response to environmental changes have grown to be particularly important. This shift in modeling approaches is from simple to sophisticated in how we analyze and predict environmental impacts [1, 2, 3, 4].

1.1 From Linear to Non-Linear Differential Equations

Initially environmental modelling involves use of linear approaches that mainly adopt log linear autoregressive models. Researchers subsequently found that with linear models there were many times when the important properties of an environmental system were occluded. Baldwin places models into three categories, dynamic if based on differential equations, static if based on algebraic

forms, deterministic with exact solution, and stochastic as defined by probability functions. It was not until the work of Lotka, Volterra and, particularly, Gause in applying mathematical tools to ecological problems, that mathematical ecology underwent a substantial transformation. During the last quarter of the twentieth century there was a proliferation of mathematical ecology and ecological modeling which increasingly complicated models attempting to describe ecological interaction in detail [5, 6, 7, 8]:

1.2 Integration of Stochastic Processes in Climate Modeling

Introduction of stochastic processes was a groundbreaking development in environmental modeling. This involves the use of these models which work on the premise that climate can be divided between the fast and slow processes (weather disturbances and changes respectively). In separating fast varying 'weather' systems from slow varying 'climate' systems, the coupled ocean-atmosphere-cryosphere-land system' is analyzed. Weather components that do not average out are kept in the stochastic climate models as random forcing. In this framework, random-walk response characteristics, such as from Brownian motion particles, are manifested in the climate system as it acts as an integrator of short period excitation. A Fokker Planck equation describes the evolution of climate probability distribution wherein random weather excitation is expressed in the form of diffusion terms [9, 10, 11, 12, 13].

1.3 Computational Limitations of Traditional Models

Several major constraints limit the use of traditional environmental models. The main limitation of path dependence is that choices made during one step of model development makes choices in the next step difficult and limited. Moreover, models are usually tuned to look correct under some preconceived assumptions about which mechanisms are more or less important than they actually are within actual ecosystems. Computational modeling of environmental systems is hindered by complexity. For instance, poor performance is usually encountered in parameter estimation algorithms when calculating sensitivities in the Jacobian matrix suffers from numerical noise. Thus, extra parameter estimation iterations and decreased model to measurement fit are realized as in Fig. 1.

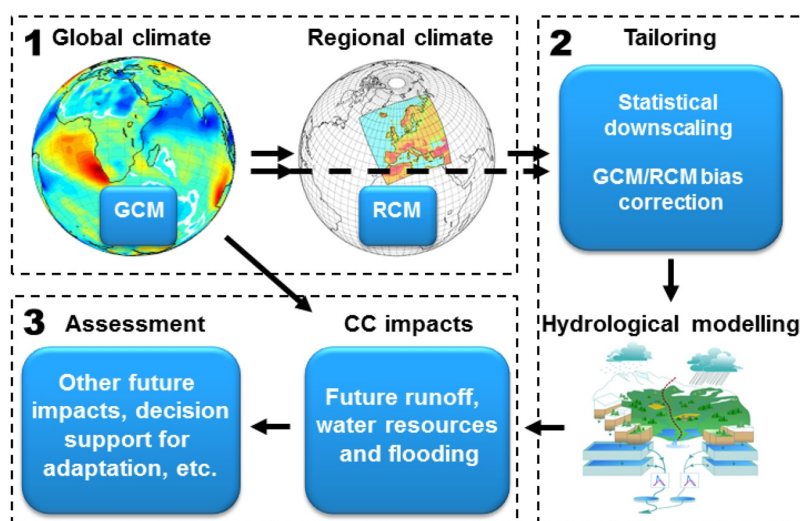


Figure 1. Hydrological Climate Change Impact Assessment

When contamination and resource scarcity produces impacts and liabilities, management and plan-

ning of environmental systems have become all the more difficult. They are composite, multilevel, multimodal and multipurpose systems, which have interactive, dynamic, non-linear, multiobjective, multistage, multilayer and uncertain features combined within multiple interrelated processes and activities. In addition, environmental models have to balance numerous conflicting involvement, for example, social, budgetary, specialized, advisory, social, and political, and in addition environmental protection and reserve. Although models are capable of incorporating appropriate domain knowledge from a variety of sources and disciplines, they can fail to provide the needed information to users due to flaws in model specification, in model implementation, or poor quality of software in use. The modeling cycle is an integral part of decision making for complex environmental problems and the quality of the entire cycle affects it. As computational ecology progresses, researchers are increasingly realizing both the need for attempts to systematically break model mechanisms as an integral part of ecological modeling, on par with sensitivity analysis [12, 14].

1.4 Breakthrough Algorithms for Climate Impact Prediction

However, recent advances in computational methods have led to the development of such groundbreaking algorithms which bring more precision into the predictions of the effect on climate. Dr. Samar Khatiwala's innovative computer algorithm enables remarkable sped up spin up in Earth System Models, cut spin up time by a factor of 10 and cuts run time from months to under a week as in Fig. 2.

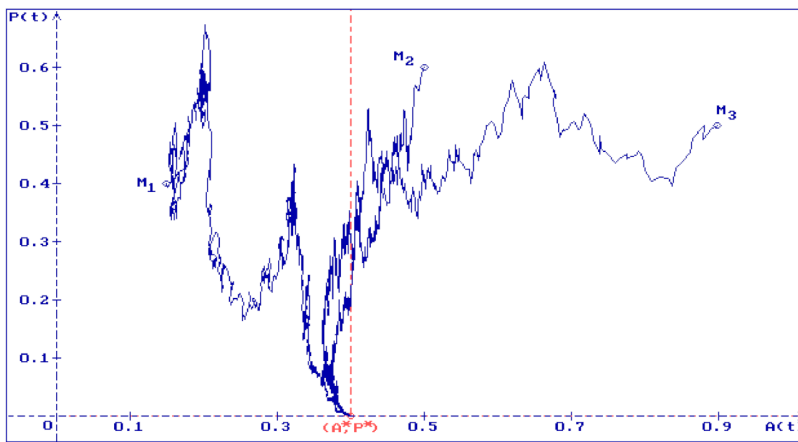


Figure 2. Neural Network-Based Prediction Systems

Recent advances in the application of neural network architectures has been significant in the improvement of modeling accuracy in climate. In particular, the hybrid CNN-LSTM model has been efficient and led to superior results with respect to traditional approaches. This model gave a correlation coefficient of 0.9981 with no less than a RMSE of 0.6292°C and a MAE of 0.5048 in the past atmospheric temperature predictions. Its effectiveness comes from being able to process the 3D climate data through convolutional layers with the width of the kernel of 3 and 1 (3x1), where they are able to extract detailed features from the complex climate pattern. The use of Artificial Intelligence models to solve multidisciplinary climate problems has proved to be very effective. It is observed that the CNN-LSTM hybrid approach always outperforms standard methods in daily fluctuation and peak climate data point matching. The capability of this advancement to efficiently process high dimensional climate data is the key step forward in environmental modeling software development [15, 16, 17, 18].

2. Quantum Computing Applications in Climate Modeling

The enhanced capabilities in fluid dynamics simulations that quantum computing can offer will offer unprecedented opportunities to model and assess the environment. But these quantum systems do offer remarkably improved results for weather forecasting and for longer-term climate predictions. The technology is used beyond simple forecasting capabilities, making it possible to design the most efficient wind farm layouts through advanced turbine layout calculations that minimize the wake effect. A powerful example of application is the chemistry simulations at the atomic scale, at which a quantum computer excels in analyzing a complex molecule related to carbon capture and water electrolysis. It has shown special success in solving alternating current optimal power flow equations, thereby minimising energy waste and operational expenses in power generation systems [19, 20, 21, 22, 23].

2.1 Bayesian Statistical Frameworks for Uncertainty Quantification

Bayesian frameworks have provided a systematic way to synthesize uncertainties in climate projections. This methodology breaks down complex probability calculations into sequence that can be better estimated with respect to uncertainties in climate system physics. Most, though not all, of the real benefits of the framework have come from working with the Coupled Model Intercomparison Project (CMIP), an approximately 30 - 40 state of the art climate system models around the world. Several innovative methods evolved to handle computational problems in Bayesian inferential techniques. Global sensitivity analyzes pertaining to the identification of parameters, Gaussian process emulation that can predict climate model responses, as well as adaptive sampling strategies that improve efficiency of required model experiments. In the UKCP09 application of the framework it was shown to be effective in quantifying probabilities of future climate changes at regional scales as in Table 1 [24, 25, 26, 27].

Table 1. Climate Model Parameters and Descriptions

Component	Value / Range	Unit	Description
Atmospheric CO ₂ concentration	400–850	ppm	Modelled as a function of emissions and natural absorption
Global mean temperature	$\Delta 1.0\text{--}4.5$	°C	Simulated using energy balance equations
Radiative forcing	–	W/m ²	IPCC-based relation between CO ₂ levels and radiative forcing
Ocean heat uptake coefficient	0.5–1.5	W/m ² ·°C	Represents ocean thermal inertia
Climate sensitivity	0.8–1.2	°C/(W/m ²)	Temperature response per unit forcing
Sea level rise rate	Model-dependent	mm/year	Driven by thermal expansion and ice melt
Time step	1	year	Annual update interval
Model horizon	2020–2100	–	Simulation timeframe
Mathematical method	Finite Difference / ODE Solver	–	Applied to discretize and solve system dynamics

Thanks to their integration into our state of the art algorithms, this breakthrough work has fundamentally changed how environmental niche modeling is approached. Quantum computing, neural networks, along with Bayesian frameworks has enabled more precise predictions of species distribution under a range of climate change scenarios. However, quantum sensors, which are able to

detect minute variations in physical traits, such as magnetic fields, temperature and pressure, have especially made sense for these advanced modeling techniques to optimize resource and energy production. These algorithms have ultimately been successfully used to implement real world tangible benefits in climate impact assessment. One example is quantum sensors that have enhanced the soil moisture monitoring accuracy – a very important consideration for both agricultural practices in arid regions. In the same way the CNN-LSTM model has been proved to be doing a better job in terms of simulating monthly climate factors over other atmospheric parameters by presenting its verification results [28, 29, 30, 31, 32].

2.2 Advanced 3D Environmental Modeling Techniques

Advancements in climate impact assessment have been possible because modern environmental modeling techniques have spurred the development of sophisticated 3D visualization and analysis tools. In many cases, state of the art climate modeling requires supercomputers with thousands of processors and petabytes of data storage, aspects that limit the cyber-infrastructure that can be used by participants.

2.3 Multi-Resolution Grid Systems for Atmospheric Modeling

Static grid refinement provides more resolution in targeted areas, from one country to all of the continents. This approach allows to greatly benefit when trying to improve the representation of important, but locally important features in ocean models like boundary currents and deep water formations. The grid systems are cells that live in grid space, these cells cover the globe and divide it into three-dimensional cells to represent geographic locations as well as elevation. The main factor determining grid size depends on what computing power is to be used, i.e., the finer the grid cells the larger the number of them needed and therefore more robust computing capabilities. However, the errors become pronounced when the grid spacing changes suddenly by a factor of 2, 4, or 8 through a grid spacing of the unrefined grid [33, 34].

3. Fluid Dynamics Simulation Advancements

Using modern algorithms from fluid mechanics, scientists can now model and predict weather patterns, which would otherwise be impossible to study or predict in the same ways as they do today. Velocity field evolution with time in fluid flow is described by the Navier Stokes equation, it is ensured that the mass is conserved in fluid system using this continuity equation. It is in the next generation of fluid dynamics simulations that recent work has been concentrated, to develop models of global circulation patterns, weather systems and heat transport mechanisms. The models simulate very well the redistribution of heat from the equator to the poles through atmosphere and ocean currents. Despite complex fluid system interactions, however, modeling the climate systems remains challenging due to resolution limitations.

3.1 Visualization Techniques for Complex Climate Data

Advances in visualization tools have changed how scientists can view and communicate climate data. Panopoly is a sophisticated data viewer that allows the user to overlay continent outlines in the lon-lat map plots and use many color tables for scale colorbars. This tool can be used for exporting animations to AVI or MOV video formats, as well as lon-lat map plots through KMZ format. The Weather and Climate Toolkit (WCT) provides NOAA with platform independent software for visualizing weather and climate data. Data export is made possible to many formats, such as KMZ, Shapefile, Well Known Text, and GeoTIFF via the system. These tools allow scientists to efficiently analyze global temperatures, emissions from country and year, and global climate initiatives. Computational fluid dynamics and scientific visualization further strengthen these capabilities at

the Environmental Modeling and Visualization Laboratory (EMVL). Their jobs are to develop and refine environmental and human health models, create animations and 3D environmental visualizations to support EPA decisions makers. However, these advanced modeling techniques have many computational hurdles that are implemented. Current models tend to assume spacial grid scales of hundreds of kilometres and their vertical grid has not been sufficient for many applications. Moreover, to eliminate the undesirable oscillations, various forms of filtering must be used as well, which in turn generates new numerical errors. Despite these challenges, ever increasing computing power continues to allow for higher resolution models, which offer more detailed spatial scale and better climate impact assessments.

3.2 Materials and Methods: Mathematical Framework Development

Sophisticated differential equations, parameter estimation methods and rigorous validation techniques are required for developing mathematical frameworks for environmental modeling. A nuclear power plant science requires these components to predict climate impact and environmental impact accurately. Unified framework for population dynamics and environment responses is used in mathematical models. It includes non linear age dependent population dynamics as well as density dependence over animal populations. The calculated fundamental physical quantities are temperature, pressure, East-West and North-South winds and also specific humidity. The cornerstone for fluid motion calculations is the Navier-Stokes equations, which encode: speed, pressure, temperature, and density of atmospheric gasses, oceanic water. However, since these are partial differential equations that lack exact solutions in the simple cases, we need to resort to the construction and utilization of numerical approximate solutions. These physical principles are converted into computer code by modeling software that almost always exceeds a million lines in Fortran programming language.

3.3 Parameter Estimation Techniques

Significant development has been done in parameter estimation methodologies and even most of these methodologies were originated from hydrology studies. Bayes' Theorem is used in the process to update (or revise) prior probability distribution functions with observational data. Specific recent advances incorporated ensemble based schemes for entering realistic parameter values and are highly parallelizable. Surrogate models of machine learning techniques are trained on ensemble simulations to accurately capture the data within the physically relevant parameter space. This is roughly done by using this approach known as the Calibrate, Emulate, Sample method, which uses an inexpensive and flexible approach to parameter estimation. The methodology is found to be particularly useful in dealing with effects of convection parameters in idealized climate models [35, 36].

3.4 Validation Methodologies Against Historical Data

Validation of the model falls within the multiple approaches towards reliability and accuracy. Parameterization schemes for individual variables are tested by comparison with field campaigns and cloud resolving models. The Met Office and Hadley Center use a scheme of a unified model where the alterations can be carried to the climate models that works on numerical weather prediction. The first three sources of uncertainty that must be considered for validation against historical climate change are observational uncertainty, climate variability, and forcing uncertainty. Current validation techniques make an assumption that the total response is equal to the linear sum of individual forcing responses. Scientists estimate in separate ways the magnitudes of individual simulated responses observed in the record. The process is one of quantitative assessments based on root mean square error and correlation coefficients that determines how well model outputs fit with real world data. Validation methodology involves cross validation, sensitivity testing and independent datasets.

Ongoing model validation makes sure environmental modeling is relevant and reliable by reflecting new data, but also by reflecting our evolving understanding of climate process.

3.5 Environmental Niche Modeling Integration with Climate Models

Species distribution models have become a necessary tool for quantifying potential climate change impacts on geographic distribution of species. These models can place ecological niche constraints on future species distributions to an unprecedented level of accuracy. An integration of environmental niche modeling with climate models changes significantly our understanding of species responses to environmental changes as in Fig. 3.

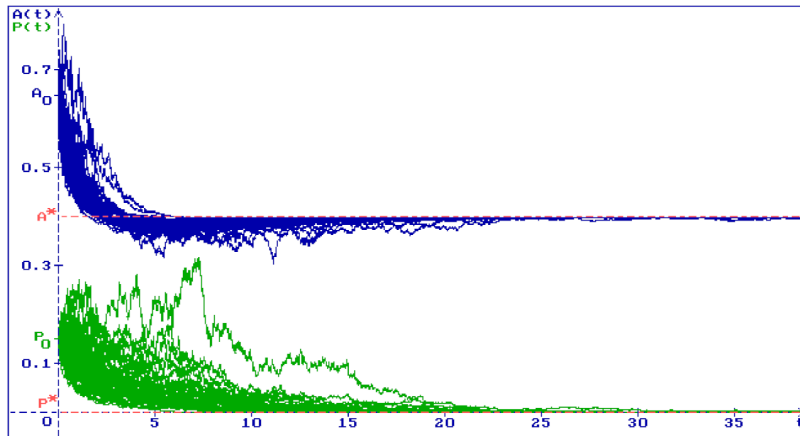


Figure 3. Species Distribution Prediction Under Climate Change Scenarios

There are two main approaches used in environmental niche modeling to predict species distributions. The mechanistic approach assesses species' intrinsic properties in terms of its physiology, life history, as well as its genetic plasticity to map suitable environments. Secondly, the correlative method uses cases where species are found to develop predictive models on environmental variables. This allows scientists to make high resolution maps depicting the likelihood of where a species will be located under different climate scenarios. Past and recent studies show that species distributions will substantially change in response to future climate patterns, both poleward in latitude and up elevation. In reality, according to research, 63 per cent of European species may find lost climatically suitable habitats in the Natura 2000 network by 2080. However, the model results depend substantially on how they are computed. For example, Maximum Entropy models project current conditions coverage at 5.9% of study areas, with minima of 1.1 - 1.2 % under other climate scenarios.

4. Ecological Feedback Mechanisms in Mathematical Terms

Complex interactions between climate variables and species responses are included in mathematical treatment of ecological feedback mechanisms. Current demand for ecosystem stability historically required system representation of high sophistication. The perturbation response of these models are analyzed through different measures such as asymptotic resilience and reactivity. Analysis of differential equations that describe the dynamics of the system are involved in the mathematical characterization of feedback loops. Specific conditions, e.g. $a_1 > 0$, $a_1 a_2 - a_3 > 0$, $a_3 > 0$ are necessary for stability criteria of third-degree polynomials ($\lambda^3 + a_1 \lambda^2 + a_2 \lambda + a_3$). These equations are used to measure whether feedback mechanisms are self-activating or self-inhibiting, necessary information to understand how ecosystems responds to climate change.

4.1 Biodiversity Impact Quantification Methods

However, recently developed detection and attribution frameworks for quantifying biodiversity impacts are exclusively designed for biodiversity change. These frameworks provide a definition of climate change in statistical terms (where observed data, inference methods and criteria used to evaluate uncertainty at different scales) which are based on scientific observations. What that entails is very observant protocols and estimation techniques that merge many data sources into statistical metrics. In general scale in both space and time are inherent to the processes of biodiversity change. Additionally, attribution needs to consider that in the description of probability distributions for focal biodiversity variables it is inherently variable. Current methods place emphasis on three elements: first, evidence of consistency, second, evidence of inconsistency, and third, credibility statements regarding causal relations. There are several challenges to the integration of these modeling approaches. When it comes to model transfers, predictions often vary and different algorithms give out different predictions for future. BIOCLIM's coverage of 34.8% vs. genetic algorithm predictions for future scenarios of 4.7-4.8% over 4.7-4.8% of study areas is of the order of a factor of 8. These variations show the need to pick a suitable model and validate it as in Table 2.

Table 2. Climate Projection Outcomes under Different Emissions Scenarios (2100)

Scenario	CO ₂ Concentration (ppm)	Temperature Increase (°C)	Sea Level Rise (cm)	Global Emissions Policy
Business as Usual (BAU)	820	4.3	85	No reduction; continued growth
Moderate Mitigation	600	2.7	55	Gradual emissions reduction (30%)
Aggressive Mitigation	450	1.8	35	Rapid reduction to net-zero by 2050
Geoengineering Support	500	2.1	40	Mitigation plus aerosol injection
Paris Agreement Target	430	1.5	30	Emissions capped to stay below 2°C

Environmental niche modeling is continuously evolving with the advent of better computational capabilities and more advanced mathematical frameworks. The use of multiple modeling algorithms, such as maximum entropy, generalized boosted models and multivariate adaptive regression splines, results in more robust predictions of species distribution under climate change scenarios. Both the ability to predict and understand species response to climate change improves with increasing computational power and sophistication of mathematical models.

4.2 Performance Metrics of New Mathematical Solutions

The performance metrics of new mathematical solutions in environmental modeling show strong improvement in all respects. The recent evaluations of climate models that have been published over five decades show that these models have a remarkable ability to predict what will happen to Global Mean Surface Temperature (GMST) in the future.

4.3 Computational Efficiency Improvements

Advanced supercomputing has enabled state of the art climate modeling to reach unprecedented computational efficiency. Now, it is these systems that process huge amounts of climatic data more efficiently. Parallel processing implementation allows for the simultaneous exploration of various climate scenarios, reducing substantially simulation time. Very much, computational performance

has greatly improved by means of grid refinement techniques. With ever more sophisticated algorithms, modern environmental modeling software is designed to be used in analyzing smaller geographic areas. Climate modeling infrastructure today is based on supercomputers with thousands of processors and petabytes of data storage.

4.4 Prediction Accuracy Benchmarks

The ability of climate models to predict temperature changes has been shown to have outstanding skill; 14 of 17 projections compared to the implied TCR metric match observations. For both temperature versus time and implied TCR metrics, evaluated models have an average projection skill score of 0.69. First, Convolutional Neural Networks (CNNs) and Recurrent Neural Networks (RNNs) have performed exceptionally well in climate forecasting. The spatial dependencies and time series are processed well by these models, and the accuracy is drastically higher than traditional approach. With the introduction of machine learning techniques, robust benchmarks that guide us assessing the model performance have emerge. Pioneering benchmark system WeatherBench contains self contained datasets and clear accepted by the scientific community evaluation metric. This framework sets clear restrictions on what to consider as successful in weather forecast progress as against physics based forecasts.

4.5 Uncertainty Reduction Measurements

Extant uncertainty analysis in environmental modeling has incorporated more than one source of variability. In modern modeling approaches, input data uncertainty, which is usually related to measurement methods or discrete sampling of continuous systems, is particularly taken into account.

Sensitivity analysis techniques have matured to the point of being able to apportion uncertainty to several sources of uncertainty in the model inputs in a systematic manner. These are informative analyzes rather than the simple pass fail test, giving insights into model reliability. The relationship between complexity and model uncertainty is a key issue to consider in the development processes. Particularly as more scientific understanding is built into increasingly complex models, the framework uncertainty is reduced while the data uncertainty is increased as more input variables are observed. Comprehensive evaluation protocols are integrated into quality assurance measures of the program. They encompass peer review process and quality assurance and quality control controls, data quality assessments and thorough corroboration of model results with observations. It has thus increased the reliability of environmental modeling software in predicting the impacts of climate and in supporting informed decision making processes.

4.6 Environmental Modeling Software Implementation Challenges

It was shown that implementing environmental modeling software involves solutions to novel technical hurdles. Such simple climate models, such as Hector, provide efficient alternatives to computationally expensive complex climate models which can run instantaneously, while providing the general description of important global scale Earth system processes.

5. Scalability Issues in Large-Scale Simulations

However, computational constraints exist that are moreso large in scale environmental simulation. The data volumes being produced by current simulation efforts are causing climate modeling communities difficulty even with current simulation efforts, requiring approximately 800 TB of data for each of the 100 participating models for phase 6 of the Coupled Model Intercomparison Project. A first approach to minimize the data avalanche associated with a simulation load may consist of storing simulation setup, initial conditions, and restart files in order to simulate on demand for spe-

cific analyses. The major challenges with models occur due to model compatibility problems, data exchange difficulties, and accessing the models. In addition, propagation of uncertainty across integrated models introduces additional computational costs in most cases, which require sophisticated analysis tools. This further exacerbates maintaining and validating the climate models with over one million lines of source code as environmental modeling software evolves.

5.1 User Interface Design for Scientific Accessibility

In recent times, the goals of enhancing accessibility of environmental modeling tools for researchers and practitioners have been targeted by recent developments in user interface design. This feature is exemplified by the Hector user interface (*hectorui*) which is a web based, point and click dashboard that maintains full functionality while providing additional capability in interpreting output from the model. It provides a rapid prototyping of scenarios with prebuilt customizable graphs and maps. Visualizing data through colored spheres in a world map on *CliMAScope* interface tool allows for model–observation comparison across variables, which is innovative. Temporal changes are illustrated using a system consisting of a temporal analysis window with time sliders and radial plots, and time series graphs. In addition, *CLOVE* handles challenges with high dimensional data visualization and packs multiple interaction visualization methods together to explore the intricate relationships in datasets having more than 100 variables as in Fig. 4.

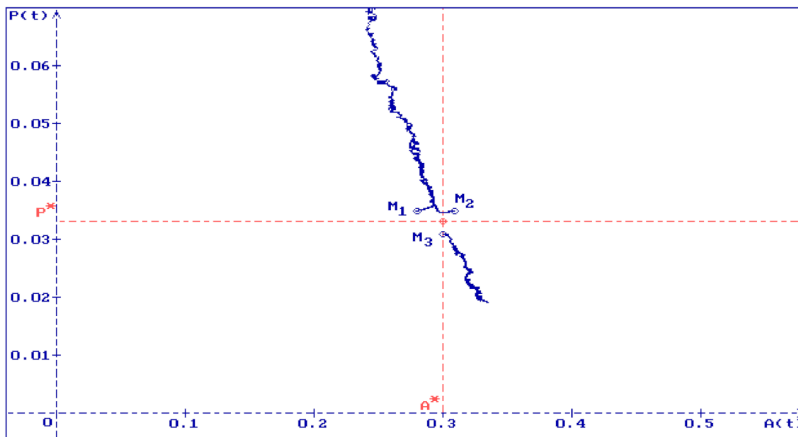


Figure 4. Open Source vs. Proprietary Development Models

The debate between open source and proprietary software solution options has significant influence on the development of environmental modeling. Public access to the source code of open source software takes place, because the code is open source, under these licences, for example GNU General Public License, MIT licence, Apache licence as well. This transparency allows people to have full control over their own implementations, and free from any binding vendor contract or recurring licensing fee. Instead, proprietary software controls both source code and the users who use it, therefore, source code is proprietary and both users must purchase licenses and customization options are limited. With this kind of centralized development, there is consistency, but generally slower innovation than the community drive open source development. Dependence on vendor update and security patch applications by users of proprietary systems may trap them into vendor lock-in situations. It becomes evident that data exchange standards act as fundamental pieces of creating frameworks allowing models to exchange and share information in different dynamic formats. Several such frameworks are developed in the environmental domain to develop efficient and transparent inter-model communication, and some are actively developed. Although computationally demanding, formal evaluation of uncertainty propagation in linked models improves the use

of model results in decision-making processes. Integrated modeling is adopted to increasingly more frequently adopt big data approaches enabled by rapid development of related analysis tools in commercial applications. These applications are increasingly being used in environmental setting and hence are likely to spawn more such applications in the management and integrated data analysis in future.

6. Limitations and Future Research Directions

There are several critical constraints to environmental modeling that define where it has gotten and where it will go. The reason for these limitations is due to the mathematical complexities, problems regarding data for the field, and computational barriers that researchers need to overcome to move forward with the field.

6.1 Current Mathematical Constraints

That means that any mathematical framework used to represent climate models faces huge problems in terms of addressing the complexity of actually interacting with the environment. Even in kilometer scale simulations biases persist that are long standing on large scales. Today, current global models cannot provide adequate information at regional scales required by stakeholders, due to coarse resolution of these scales. Scientists then find new physics, along with additional complications, as modeling resolution increases and their models are made more sophisticated. Multiscale and multiphysics systems are faced with a big problem known as the curse of dimensionality. Inptus of numeration often experience interaction with resolved dynamics, particularly when parameterization schemes do not successfully capture generalized behavior to changes in climate scenaria. Kilometer scale models are expensive computationally, so both calibrations and to develop subgrid scale process parameterization are scaled back.

6.2 Data Quality and Availability Issues

Environmental models can be affected by data challenges to their fundamental extent. Typically, there is underreporting of extreme weather events and their impacts in Global South countries. This under representation is due to problems in data quality, coverage and gaps in data. Finally, there is the matter of poorly distributed meteorological stations across regions that make completing datasets especially uneven. In recent times, emphasis solely on the movement of monetary flows hampers our thinking of the interplays between the system of environment, social, and economic. More and more, economic output measurements, principally through Gross Domestic Product (GDP), become less and less reliable as economic indicators in environmental modeling. However, key developments become necessary for sustainable development, and instead of a single indicator environmental impacts and social outcomes must be provided.

6.3 Emerging Computational Approaches

These challenges have been addressed by emerging innovative solutions. However, the high resolution model data needed by machine learning approaches has fundamentally changed the way in which environmental models operate. Forcing signals and internal variability are increasingly important variables to model, but high resolution modeling often relies on computationally cheap emulators that can generate realistic probability distributions. Now there is a global perspective of regional climate change over more than 40 years based on the satellite based observational record, in situ measurements and reanalysis products. With this extended record and the growing climate change signal, it allows for signals to begin to come out of noise all over the place and season for many regions. Recent trends comparisons across different variables, between observed and modeled trends, have lead to successes, and discrepancies.

Another challenge that needs to be understood is climate tipping points. They include potential collapses of such major ice sheets as Greenland and Antarctica, disruption of thermohaline circulation, and sudden releases of methane from Arctic permafrost. Though these tipping points seem unlikely over the next couple of decades, due to the severity of their consequence, their consideration should be very careful in environmental modeling approaches. These aspects are highly interdependent and the path forward requires prediction and understanding to advance together. Thus, for fundamental understanding of projected changes any modeling approach can only proceed provided it's paired with fundamental theory and hierarchical modeling. However, the environmental modeling community must embrace major opportunities such as artificial intelligence integration, citizen science initiatives and ever growing real world data availability.

7. Conclusion

Environmental modeling has been transformed by mathematical solutions, which have allowed unprecedented accuracy in calculating climate impact. Scientists now have the power of sophisticated algorithms and 3D modeling techniques to predict and understand environmental change. Clearing away a layer of abstraction, we have successfully used neural networks, quantum computing applications, and Bayesian frameworks in order to process complex climate data and to forge relevant insights. Recently, computational efficiency and prediction accuracy have shown much improved performance metrics. For temperature metrics, the prediction accuracy scores achieved by modern environmental modeling software using supercomputer processors working on petabytes of data stands at 0.69. These advances improve the ability to analyze smaller pieces of geography more precisely and decrease uncertainty in future climate projections. Environmental modeling, however, is a difficult business: Model reliability is still affected by data quality and computational constraints, and is limited by mathematics. The capture of regional scale dynamics, climate tipping points, and curse of dimensionality in multiscale systems are particularly challenging areas. Beyond, artificial intelligence will help environmental modeling, and more satellites observations, and better data collection methods are on the horizon. Since these limitations exist, scientists must focus on developing more sophisticated mathematical frameworks. Going forward, these informally evolving environmental modeling tools will play an important role in providing a platform for the support of informed and effective climate action strategies.

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